# TECHNICAL NOTE

D-990

# METHODS FOR DETERMINING THE OPTIMUM DESIGN OF STRUCTURES PROTECTED FROM AERODYNAMIC HEATING AND APPLICATION TO TYPICAL BOOST-GLIDE OR REENTRY FLIGHT PATHS

By Robert S. Harris, Jr., and John R. Davidson

Langley Research Center
Langley Air Force Base, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON March 1962

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-990

# METHODS FOR DETERMINING THE OPTIMUM DESIGN OF STRUCTURES PROTECTED FROM AERODYNAMIC HEATING AND APPLICATION TO TYPICAL BOOST-GLIDE OR REENTRY FLIGHT PATHS

By Robert S. Harris, Jr., and John R. Davidson

## SUMMARY

General equations are developed for the design of efficient structures protected from thermal environments typical of those encountered in boost-glide or atmospheric-reentry conditions. The method is applied to insulated heat-sink stressed-skin structures and to internally cooled insulated structures. Plates loaded in compression are treated in detail. Under limited conditions of plate buckling, high loading, and short flight periods, and for aluminum structures only, the weights of both configurations are nearly equal. Load parameters are found and are similar to those derived in previous investigations for the restricted case of a constant equilibrium temperature at the outside surface of the insulation.

## INTRODUCTION

An aircraft which flies at high speeds in, or reenters into, an atmosphere generates severe thermal environments by its high-speed motion. Elevated temperatures have a deleterious effect upon such material properties as the allowable yield stress and Young's modulus; and, since the structural strength depends upon these properties, the amount of load that the structure can support decreases as the temperature rises. (In rare instances mild heating might increase an allowable yield stress, but this behavior is not utilized in design.) The high temperatures associated with the thermal environment can raise the temperature of the aircraft structure to a point where the material has insufficient strength to support the aerodynamic loads. Structure temperatures can be reduced by protecting the structure from its thermal environment through the use of insulation or cooling, or both, thus permitting lighter primary structures. However, the heat-protection system itself adds weight to the aircraft and must be considered when the weight of the protected structure is calculated.

For the purposes of this report, an optimum structure is defined as the lightest possible structure which supports a given load within a specific thermal environment. In obtaining the structural weight the weight of the thermal protection system is included. Hence, a protected structure is efficient if the combined weight of the protected structure and protection equipment is less than the weight of an unprotected structure designed for the same flight conditions.

References 1 and 2 consider insulated heat-sink structures wherein the structure itself is used as a heat sink, and insulation is placed between the high-temperature atmosphere and the structure to retard the flow of heat. Throughout those analyses the effective environmental temperature was considered constant. It was found that the system weight could be expressed as a function of structure temperature. An optimum structure was determined by minimizing the weight with respect to the temperature. From the results load parameters were derived through which it was possible to plot the weights of optimum configurations. The load parameters include the applied load, the flight time, and the insulation characteristics. There was a load parameter associated with each of the design criteria of yield, buckling, and post-buckling failure. The minimization of the weight with respect to the temperature works well for the case of constant environmental temperature, but involves an excessive amount of algebraic manipulation when applied to problems where the thermal environment varies with time.

L
9
9
1

A more general thermal environment, which is typical of that experienced by a boost-glide missile or atmospheric-reentry body, is considered herein. The work of the previous reports is extended to include insulated and internally cooled structures as well as the insulated heat-sink configurations. The algebraic difficulties of the parametric method are circumvented by applying Lagrange's method of undetermined multipliers. The results lead to load parameters very similar to those of the case of a constant-temperature environment. Computations are made for a water-cooled structure, and the results are compared with those for an insulated heat-sink structure on the basis of plate buckling strength.

## SYMBOLS

| | |
|---|---|
| A | constant, $^{\circ}F/(sec)^n$ |
| a | dummy index |
| B | exponential decay constant, 1/sec |
| b | plate width, ft |

$c_1$ heat capacity of insulation, Btu/lb-°F

$c_2$ heat capacity of metal primary structure, Btu/lb-°F

$c_3$ heat of vaporization of coolant, Btu/lb

$c_3'$ effective heat of vaporization when weight of pump and piping equipment is proportional to water required, Btu/lb

$E$ Young's modulus, lb/sq ft

$e$ exponential base, 2.718

$$G = \int_{\tau_j}^{\tau_k} \left(T_{eq} - T_v\right) d\tau$$

$g\left(t_1, t_2\right)$ function of $\left(t_1\right)$ and $\left(t_2\right)$

$H = BG$ (see eq. (13))

$h$ heat-transfer coefficient, Btu/sq ft-sec-°F

$K$ buckling coefficient

$k$ thermal conductivity, Btu/ft-sec-°F

$n$ constant

$P$ load, lb

$Q$ amount of heat absorbed by coolant, Btu/sq ft

$q\left(t_1, t_3\right)$ function of $\left(t_1\right)$ and $\left(t_3\right)$

$T$ temperature, °F

$t$ thickness, ft

$u,x,y$ dummy variables of integration

$W$ weight per unit area, lb/sq ft

$w$ total weight per unit area of pump and piping equipment when distributed over heated surface of vehicle, lb/sq ft

$w_p$ weight per unit area of fixed weight portion of pump and piping equipment when distributed over heated surface of vehicle, lb/sq ft

$\alpha = \dfrac{k_1}{c_2\rho_2 t_2 t_1}$, 1/sec

$\beta$ exponential decay constant (associated with structural material properties), $1/{}^{\circ}F$

$\gamma$ constant (associated with structural material properties), lb/sq ft

L
9
9
1

$\epsilon$ ratio of variable portion of pump and piping weight to coolant weight (see eq. (32))

$\lambda$ nondimensional time parameter, $\alpha\tau$

$\mu$ Poisson's ratio

$\rho$ density, lb/cu ft

$\sigma$ general material strength property or parameter, lb/sq ft

$\sigma_y$ yield stress, lb/sq ft

$\tau$ time, sec

$\tau_{cr}$ critical time, sec

$\psi$ Lagrange undetermined multiplier

$\Omega_{b,e}$ load parameter for buckling, $\left(\dfrac{P}{b^2}\right)^{2/3} \dfrac{Bb^2}{k_1\rho_1}$, $\left(\dfrac{lb}{ft^2}\right)^{2/3} \left(\dfrac{Btu}{ft^3\text{-}^{\circ}F}\right)^{-1} \left(\dfrac{lb}{ft^3}\right)^{-1}$

$\Omega_{y,e}$ load parameter for yield, $\left(\dfrac{P}{b^2}\right)^{2} \dfrac{Eb^2}{k_1\rho_1}$, $\left(\dfrac{lb}{ft^2}\right)^{2} \left(\dfrac{Btu}{ft^3\text{-}^{\circ}F}\right)^{-1} \left(\dfrac{lb}{ft^3}\right)^{-1}$

Subscripts:

1 insulation

2 primary structure

3 coolant

aw adiabatic wall (temperature)

L991

b buckling

eq equilibrium temperature

i indicial notation

j,k signify beginning and end of coolant vaporization period

max maximum

o initial conditions

v vaporization

y yield

## ASSUMPTIONS

A sketch of the insulated heat-sink stressed-skin structure is shown in figure 1. Figure 2 is a sketch of the insulated and cooled structure. The primary structures are considered to be edge-supported plates loaded in compression such as covers of a box beam, or some other load-carrying surface exposed to aerodynamic heating. The volume of coolant required per unit of heated surface area is denoted by a "thickness" $t_3$. The subscripts 1, 2, and 3 have been assigned to the insulation, primary structure, and coolant, respectively.

The following assumptions have been made to simplify the analysis:

(1) The heating is uniform over the entire surface of the member; that is, the heat flow is one dimensional.

(2) The primary structure supports the entire load.

(3) The temperature gradient through the metal skin is negligibly small.

(4) For the case of the heat-sink structures the thermal capacity of the insulation can be neglected with respect to the thermal capacity of the primary structure (metal skin). This approximation may be improved by the iteration method of appendix A of reference 1. The iteration method is valid for cases where $c_1\rho_1t_1 < 2c_2\rho_2t_2$ and should be used whenever $\frac{c_1\rho_1t_1}{c_2\rho_2t_2} > \frac{1}{20}$. For the internally cooled structure it is assumed that all of the heat passing through the outside surface of the insulation is absorbed by the coolant.

(5) The outside surface temperature of the insulation $T_{eq}$ is a known function of time. This surface temperature can be determined by the method of appendix B of reference 1. L991

(6) The heat capacity and thermal conductivity of the materials are independent of temperature. (Note that the yield stress and Young's modulus are considered functions of temperature.)

(7) The coolant used in the insulated and cooled structure absorbs heat by vaporizing. The thermal capacity obtained by raising the coolant temperature from $75^{\circ}$ F to $212^{\circ}$ F is neglected.

Assumption (4) has the greatest effect upon the accuracy of the analysis of the insulated heat-sink configuration. For the ranges of thermal capacities commonly encountered in practice, the assumption will usually lead to acceptable engineering accuracy without iteration. This accuracy is evaluated in reference 3.

Assumption (7) will lead to a conservative (high) answer for the weight of the insulated and cooled configuration. The tendency to overestimate the coolant required is somewhat offset by considering the heating period to exist only during the period wherein the equilibrium temperature exceeds the coolant boiling temperature. This situation is discussed subsequently in more detail.

## CHARACTERISTICS OF THE ASSUMED THERMAL ENVIRONMENT

References 1 and 2 examine the problem of finding the optimum weight of an insulated structure exposed to a constant equilibrium temperature at the outside surface of the insulation. In order to optimize the weight of a structure exposed to boost-glide or reentry flight paths, it is necessary to consider a variable equilibrium temperature at the outer surface of the insulation. The development of the equations in parts of this report is general and independent of any particular time

variation of the equilibrium temperature except for the unobstructive requirement that the equilibrium temperature eventually decreases during the flight period, and that, consequently, the primary structure temperature has a definite maximum value. This mathematical restriction does not impede the solution of physically real problems concerning recoverable vehicles which eventually must slow to landing speeds and hence, as a natural result, experience a decrease in surface equilibrium temperature.

In order that demonstrative results might be obtained to illustrate the application of the optimizing relationships, the following particular time variation in equilibrium temperature $T_{eq}$ was chosen:



$$T_{eq} - T_o = A\tau^n e^{-B\tau} \tag{1}$$

The constants A, B, and n may be determined to give a best fit to some experimental or predicted surface-temperature curve of interest. Relations which may be useful for curve fitting are

$$B\tau = n \tag{2}$$

when

$$T_{eq} = T_{eq,max}$$

and

$$T_{eq,max} = \frac{A}{B^n} n^n e^{-n} + T_o \tag{3a}$$

or

$$\frac{A}{B^n} = \frac{T_{eq,max} - T_o}{n^n e^{-n}} \tag{3b}$$

Equation (1) is plotted in figure 3 for five values of n to show how the shape of the curve is modified by n. In addition, figure 4 shows by a nondimensional plot that the curve has a sharper peak as n is increased.

This particular temperature variation was chosen for a number of reasons: As seen in figure 3, the equation generates a temperature history which might be considered typical of a boost-glide or reentry heating condition. In addition, for the purposes of analysis it was desired to have an analytic function which was easily differentiable and integrable. Finally, the number of constants required to fit the curve was kept small thereby keeping the constants appearing in the loading and weight parameters small.

Reference 3 gives several solutions to the heat-transfer problem of the insulated heat-sink structure. For the purposes of this investigation the first approximation, as was mentioned previously in the assumptions, was used. Equation (34) of reference 3 gives the primary structure temperature as

$$T_2 - T_o = \left(T_{aw} - T_o\right)_{\lambda=0}\left[1 - \exp\left(- \frac{\lambda}{1 + \zeta}\right)\right] + \int_0^{\lambda} \left[1 - \exp\left(- \frac{\lambda - u}{1 + \zeta}\right)\right] \frac{d\left(T_{aw} - T_o\right)}{du} du$$



where

$$\lambda = \frac{k_1 \tau}{c_2 \rho_2 t_2 t_1}$$

and

$$\zeta = \frac{k_1}{h t_1}$$

It is shown in reference 3 that this solution for aerodynamic heating, based on adiabatic wall temperature and heat-transfer coefficient, can be converted to the solution of the problem of known surface temperature by performing the manipulation that, as $h \rightarrow \infty$, $T_{aw} \rightarrow T_{eq}$. See ref. 1 or 3 for the determination of $T_{eq}$. This manipulation results in

$$T_2 - T_o = \left(T_{eq} - T_o\right)_{\lambda=0}\left(1 - e^{-\lambda}\right) + \int_0^{\lambda} \left[1 - e^{-(\lambda-u)}\right] \frac{d\left(T_{eq} - T_o\right)}{du} du \qquad (4)$$

Equation (4) may be written as

$$T_2 - T_o = \left(T_{eq} - T_o\right)_{\tau=0}\left(1 - e^{-\alpha\tau}\right) + \int_0^{\tau} \left[1 - e^{-\alpha(\tau-y)}\right] \frac{d\left(T_{eq} - T_o\right)}{dy} dy \qquad (5)$$

where

$$\alpha = \frac{k_1}{c_2 \rho_2 t_2 t_1}$$

Through the definition of $\alpha$ equation (5) shows that the first approximation will always lead to the general result that, functionally

$$T_2 = T_2\left(t_1 t_2\right) \tag{6}$$

where $\left(t_1 t_2\right)$ always appears as a product. Because of this characteristic,

$$t_2\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1} = t_1\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2} \tag{7}$$

where the subscripts on the parentheses indicate quantities held constant during differentiation. Use will be made of this property subsequently.

L
9
9
1

Combining equation (1) with equation (5) yields

$$T_2 - T_o = \frac{A\alpha}{\alpha - B}\left[\tau^n e^{-B\tau} - ne^{-\alpha\tau}\int_0^{\tau} y^{n-1} e^{(\alpha-B)y} dy\right] \tag{8}$$

Temperatures obtained from equation (8), with $n = 2$ and $T_{eq,max} = 3{,}000^\circ$ F, are shown as a dashed line in figure 5. For the case of the heat-sink configuration the maximum temperature of the primary structure occurs at the time when its temperature equals the equilibrium temperature; thereafter, aerodynamic cooling takes place. The maximum temperature of the primary structure is the design temperature and may be found by setting the right-hand side of equation (8) equal to the right-hand side of equation (1). The result is

$$\frac{B^n}{A}\left(T_{2,max} - T_o\right) = \left(B\tau_{cr}\right)^n e^{-B\tau_{cr}} \tag{9}$$

where $B\tau_{cr}$ is determined from

$$\frac{B}{\alpha}\left(B\tau_{cr}\right)^n = nB^n e^{-\alpha\tau_{cr}}\int_0^{\tau_{cr}} y^{n-1} e^{(\alpha-B)y} dy \tag{10}$$

In equations (9) and (10) the time of occurrence of the maximum temperature $T_{2,max}$ is denoted as the critical time $\tau_{cr}$. Mathematically there are three unknowns in the two equations, $T_{2,max}$, $\tau_{cr}$, and $\alpha$. The value of $\alpha$ depends upon the insulation and primary-structure thicknesses $t_1$ and $t_2$ which are not known as yet. The additional equation required to determine the proper value of $\alpha$ is found from the condition that the structure must be an optimum. Equations (9), (10), and the optimization equation are sufficient to determine the unknowns.

In the following development the weight of an optimum heat-sink structure is compared with that of an optimum cooled structure. It is therefore desirable to be able to express the weight of the cooled configuration in terms of the same parameters as those that are obtained for the heat-sink configuration, and the following equations will be useful for the manipulations of the equation for the weight of the cooled configuration.

A result of assumption (7) is that the coolant is considered to be expended only during the period in which the equilibrium temperature exceeds the temperature at which the coolant vaporizes. The times at which the equilibrium temperature equals the vaporization temperature may be found from L 9 9 1

$$T_v - T_o = T_{eq} - T_o = \frac{A}{B^n}(B\tau)^n e^{-B\tau} \tag{11}$$

The vaporization temperature and initial temperature are known as design conditions, and the two roots of equation (11) are found as $B\tau_j$ and $B\tau_k$. (See fig. 5.) These roots are functions only of $A/B^n$, a known characteristic of the equilibrium-temperature function used herein. The amount of heat absorbed by the coolant is

$$Q = \frac{k_1}{t_1} \int_{\tau_j}^{\tau_k} \left(T_{eq} - T_v\right) d\tau$$

Because $t_1$ is not known until the optimum amount of insulation has been determined, it is convenient to define $G$ as follows:

$$G = \int_{\tau_j}^{\tau_k} \left(T_{eq} - T_v\right) d\tau \tag{12a}$$

in the general case where $G$ is proportional to the amount of required coolant. For the particular equilibrium-temperature variation considered herein, because of equation (11), the general equation (12a) is better expressed as

$$\begin{aligned} G &= \frac{1}{B} \int_{B\tau_j}^{B\tau_k} \left[T_{eq}(x) - T_v\right] dx \\ &= \frac{1}{B} \int_{B\tau_j}^{B\tau_k} \left[\frac{A}{B^n} x^n e^{-x} - \left(T_v - T_o\right)\right] dx \end{aligned} \tag{12b}$$

or

$$G = \frac{H}{B}$$

where

$$H = \int_{B\tau_j}^{B\tau_k} \left[\frac{A}{B^n} x^n e^{-x} - \left(T_v - T_o\right)\right] dx$$

L 9 9 1

$$= \left(T_v - T_o\right)\left\{B\tau_j - B\tau_k + \sum_{a=1}^{n} \frac{n!}{(n - a)!}\left[\frac{1}{\left(B\tau_j\right)^a} - \frac{1}{\left(B\tau_k\right)^a}\right]\right\} \qquad (13)$$

The second expression is obtained by successive integration by parts of the first expression.

The values of $B\tau_j$ and $B\tau_k$ are functions only of $A/B^n$ as was noted after equation (11), and thus $H$ is a function only of $A/B^n$. Because of equation (3b), $A/B^n$ is a function only of $T_{eq,max} - T_o$ and $n$, and it is assumed that $T_v$, $T_{eq,max}$, $T_o$, and $n$ are all known.

## INSULATED HEAT-SINK STRUCTURES

### General Equation

The general case of the insulated heat-sink stressed-skin structure subjected to variable temperature at the outside insulation surface (see fig. 1) is discussed first. The insulated panel is heated aerodynamically, and part of the heat entering the insulation outer surface from the boundary layer is radiated away to the surroundings. The remainder of the heat input is conducted through the insulation to the structure. Appendix B of reference 1 discusses a simple method of determining the temperature of the insulation outer surface by considering a heat balance between input and radiation. This surface temperature is called the equilibrium temperature. The problem is to find the combination of insulation and stressed-skin thicknesses which results in the lowest combined weight for a given loading and temperature environment. Because the weights of surfaces are concerned, it is only necessary to determine the minimum weight per square foot of surface to find the optimum dimensions of the insulated panel. The equation relating the weight to the thicknesses of the insulation and the primary structure is

$$W = \rho_1 t_1 + \rho_2 t_2$$

which may be written functionally as

$$g(t_1, t_2) = 0 = W - \rho_1 t_1 - \rho_2 t_2 \tag{14}$$

The assumption is made at this point that the kind of metal to be used in the primary structure and the kind of insulating material have been chosen. (The insulating material should be selected on the basis of minimum $k_1 \rho_1$ and ability to withstand $T_{eq,max}$ as is discussed in I
refs. 1 and 2.) The following physical and thermal characteristics are 9
thus fixed: $k_1$, $\rho_1$, $\rho_2$, and $c_2$. It is also assumed that the dimen- 9
sion $b$ is prescribed. (See fig. 1.) 1

The allowable load depends upon one or more material properties and the thickness of the primary structure; therefore,

$$P = P(\sigma, t_2) \tag{15}$$

where $P$ is the imposed load for which the plate must be designed. The load and plate width $b$ are prescribed in a design problem, and $T_2$ is usually adjusted to make the design load equal the allowable load. The symbol $\sigma$ stands for a general material strength property such as Young's modulus or yield stress, and is assumed to be a function of temperature alone. Thus,

$$\sigma = \sigma(T_2) \tag{16}$$

The temperature $T_2$ is that of the primary structure. For design purposes the value of $\sigma$ is determined at the maximum value of $T_2$ because the metal mechanical properties are at a minimum at the maximum temperature of the primary structure. The maximum value of $T_2$ is not yet known because the relative dimensions of the optimum structure are yet to be determined. The temperature of the primary structure $T_2$ is a function of time and the insulation and metal thicknesses; thus,

$$T_2 = T_2(t_1, t_2, \tau) \tag{17}$$

The temperature $T_2$ is also a function of the time-dependent equilibrium temperature. However, it is assumed that $T_{eq} = T_{eq}(\tau)$ is a given function, and therefore $T_{eq}$ does not appear explicitly in equation (17).

In references 1 and 2 the determination of the optimum weight was approached as an extremum problem where the weight was to be minimized

for a given imposed load. Identical results may be obtained by maximizing the load with respect to a fixed weight. Lagrange's method of undetermined multipliers is used herein, with the weight considered fixed and the load to be maximized. The thicknesses $t_1$ and $t_2$ are considered to be the variables, and equation (14) is used as the equation of constraint. The following two equations result from the application of Lagrange's method to equations (14) and (15):

$$\frac{\partial P}{\partial t_1} + \psi \frac{\partial g}{\partial t_1} = 0 \tag{18a}$$



$$\frac{\partial P}{\partial t_2} + \psi \frac{\partial g}{\partial t_2} = 0 \tag{18b}$$

where $\psi$ is the Lagrangian multiplier. The set of equations (18) is subject to the restraining condition of equation (14). Equations (18) become

$$\frac{\partial P}{\partial t_1} - \psi \rho_1 = 0$$

$$\frac{\partial P}{\partial t_2} - \psi \rho_2 = 0$$

which yield

$$\frac{\rho_2}{\rho_1} = \frac{\partial P/\partial t_2}{\partial P/\partial t_1} \tag{19}$$

The partial derivatives in equation (19) can be expanded by using the previous functional relationships with the objective of obtaining partial differential terms that may be easily evaluated from known strength and heat-transfer equations. From equation (15) (keeping in mind eqs. (16) and (17) and considering $t_1$, $t_2$, and $\tau$ as the independent variables),

$$\frac{\partial P}{\partial t_2} = \left(\frac{\partial P}{\partial t_2}\right)_\sigma + \left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\left(\frac{\partial \sigma}{\partial t_2}\right)_{t_1,\tau}$$

where the subscripts indicate variables held constant during the differentiation.

It is true that

$$\left(\frac{\partial\sigma}{\partial t_2}\right)_{t_1,\tau} = \frac{\partial\sigma}{\partial T_2}\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1,\tau} = \frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1,\tau}$$

where the last form is a consequence of equation (16). Thus,

$$\frac{\partial P}{\partial t_2} = \left(\frac{\partial P}{\partial t_2}\right)_\sigma + \left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1,\tau} \tag{20a}$$

L
9
9
1

which is a convenient form.

Similarly,

$$\frac{\partial P}{\partial t_1} = \left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau} \tag{20b}$$

Substitution of equations (20) into equation (19) yields

$$\frac{\rho_2}{\rho_1} = \frac{\left(\frac{\partial P}{\partial t_2}\right)_\sigma}{\left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}} + \frac{\left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1,\tau}}{\left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}}$$

which may be reduced to

$$\frac{\rho_2}{\rho_1} = \frac{\left(\frac{\partial P}{\partial t_2}\right)_\sigma}{\left(\frac{\partial P}{\partial T}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}} + \frac{\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1,\tau}}{\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}} \tag{21a}$$

Equation (21a) is multiplied by $t_2/t_1$ to obtain

$$\frac{\rho_2 t_2}{\rho_1 t_1} = \frac{\frac{t_2}{t_1}\left(\frac{\partial P}{\partial t_2}\right)_\sigma}{\left(\frac{\partial P}{\partial \sigma}\right)_{t_2}\frac{d\sigma}{dT_2}\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}} + \frac{t_2\left(\frac{\partial T_2}{\partial t_2}\right)_{t_1,\tau}}{t_1\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}} \tag{21b}$$

Use will now be made of the property of the heat-transfer equation exhibited by equation (7). When equation (7) is substituted into equation (21b) there results

$$\frac{\rho_2 t_2}{\rho_1 t_1} = \frac{\frac{t_2}{t_1}\left(\frac{\partial P}{\partial t_2}\right)_\sigma}{\left(\frac{\partial P}{\partial \sigma}\right)_{t_2} \frac{d\sigma}{dt_2}\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2,\tau}} + 1 \tag{22}$$

L 9 9 1

It is only necessary to substitute the appropriate partial derivatives into equation (22) to obtain the optimum conditions for an insulated heat-sink structure. Equation (22) is a general result and applies to any time variation of equilibrium temperature. Examination of the first term on the right-hand side of the equation shows that $\frac{d\sigma}{dT_2} < 0$ because material properties generally decrease with temperature, and that $\left(\frac{\partial T_2}{\partial t_1}\right)_{t_2} < 0$ because an increase in the insulation thickness will decrease the temperature $T_2$. All other terms are positive. Equation (22) thus indicates that

$$\rho_2 t_2 > \rho_1 t_1$$

or that the primary structure is heavier than the insulation for an optimum heat-sink design.

## Application of Mechanical Strength Criteria

Design on the basis of yield stress.- The optimization equation (eq. (22)) is used to determine the optimum configuration for a structure designed on the basis of yield. For the yield condition equation (15) becomes

$$P = bt_2\sigma_y \tag{23}$$

An empirical relationship that relates the yield stress to temperature was used in references 1 and 2 and will be used herein; the specific form of equation (16) to be used in the analysis is then

$$\sigma_y = \gamma_{1,y} e^{-\beta_{1,y} T_2} \tag{24}$$

This empirical representation of the yield stress is plotted in figure 6 with experimental data for 2024-T3 aluminum alloy. The functional equation (17) for the structure temperature is represented by the specific equation (8). For design purposes, however, the maximum structure temperature should be used. (This maximum is given by the simultaneous solution of eqs. (9) and (10).) Taking the appropriate partial derivatives of equations (9), (10), (23), and (24), and substituting the results into equation (19) yields

$$\frac{B}{\alpha}\left(\frac{\sigma_y}{\rho_2}\right)^2 \frac{1}{c_2}\left\{1 - \frac{\left(1 - \frac{B}{\alpha}\right)}{\beta_{1,y}\left(T_{2,max} - T_o\right)\left[\frac{B}{\alpha} - \left(B\tau_{cr} - n\right)\right]}\right\} = \left(\frac{P}{b^2}\right)^2 \frac{Bb^2}{k_1\rho_1} = \Omega_{y,e} \qquad (25)$$

L 991

where the subscript e is used to distinguish the load parameter $\Omega_{y,e} = \left(\frac{P}{b^2}\right)^2 \frac{Bb^2}{k_1\rho_1}$ from the load parameter $\Omega_y = \left(\frac{P}{b^2}\right)^2 \frac{b^2}{k_1\rho_1\tau}$, which is a result of references 1 and 2 where the equilibrium temperature was considered constant. The two parameters are the same dimensionally. In the case of constant equilibrium temperature the flight period (and hence the heating period) is known. In the variable equilibrium temperature considered herein the time of occurrence of the maximum structure temperature is not known a priori, but the value of B is known from the characteristics of the equilibrium-temperature curve.

The value of $\alpha$ for the optimum configuration can be found from the simulataneous solution of equations (9), (10), (24), and (25) (with $T_{2,max}$ used in eq. (24)). For the purpose of computing design charts it is easier to solve this system of equations inversely by first assuming values of $B/\alpha$ for fixed values of $A/B^n$, determining the values of $T_{2,max}$ and $\sigma_y$ from equations (9), (10), and (24) (or plots thereof), and then computing $\Omega_{y,e}$ from equation (25). After these final results are plotted $\alpha$ may be determined for given values of $\Omega_{y,e}$.

The substitution of equations (23) and (25) into equation (14) yields, after manipulation

$$\frac{W}{b}\left(\frac{Bb^2}{k_1\rho_1}\right)^{1/2} = \frac{\rho_2}{\sigma_y}\left[1 + \left(\frac{\sigma_y}{\rho_2}\right)^2 \frac{B}{\alpha} \frac{1}{c_2\Omega_{y,e}}\right]\Omega_{y,e}^{1/2} \qquad (26)$$

where $\sigma_y$ and $B/\alpha$ are found through charts of the solution of equation (25) by using the given value of $\Omega_{y,e}$. These values of $\sigma_y$ and $B/\alpha$ are values for an optimum structure because of equation (25).

Design on the basis of plate buckling.- For plate buckling the equation relating the applied load to the plate dimensions and material properties characterized by equation (15) is

$$P = \frac{K\pi^2 E}{12(1 - \mu^2)} \frac{t_2{}^3}{b} \tag{27}$$

where $K = 4$ for a plate simply supported along the edges. (See ref. 4.) An empirical representation for the elastic modulus is (from ref. 2)

L
9
9
1

$$E = \gamma_{i,b} e^{-\beta_{i,b} T_2} \tag{28}$$

where $\gamma_{i,b}$ and $\beta_{i,b}$ are adjusted to give good fit for two ranges of temperature. The representation is shown in figure 7 for aluminum.

Equations (9), (10), (27), and (28) are substituted into equation (22) to obtain the relationship between the buckling load factor and the optimum value of $\alpha$. Thus,

$$\frac{B}{\alpha} \frac{1}{c_2 \rho_2{}^2} \left[\frac{\pi^2 E}{3(1 - \mu^2)}\right]^{2/3} \left\{1 - \frac{3\left(1 - \frac{B}{\alpha}\right)}{\beta_{i,b}\left(T_{2,max} - T_o\right)\left[\frac{B}{\alpha} - \left(B\tau_{cr} - n\right)\right]}\right\} = \left(\frac{P}{b^2}\right)^{2/3} \frac{Bb^2}{k_1 \rho_1}$$

$$= \Omega_{b,e} \tag{29}$$

It should be noted that $\Omega_{b,e} \neq \Omega_{y,e}$ because the loading index $P/b^2$ is raised to a different power.

Substituting equation (29) into equation (14) yields, after manipulation,

$$\frac{W}{b}\left(\frac{Bb^2}{k_1 \rho_1}\right)^{1/2} = \frac{\rho_2}{\left[\frac{\pi^2 E}{3(1 - \mu^2)}\right]^{1/3}} \left\{1 + \frac{B}{\alpha} \frac{\left[\frac{\pi^2 E}{3(1 - \mu^2)}\right]^{2/3}}{\rho_2{}^2 c_2 \Omega_{b,e}}\right\} \Omega_{b,e}{}^{1/2} \tag{30}$$

which is similar in form to equation (26).

## INSULATED AND COOLED STRUCTURE

### General Equation

For the development of an optimization equation for the insulated and cooled structure, the configuration is assumed to be as represented in figure 2. The "thickness" $t_3$ represents the volume of coolant per square foot of heated structure surface and is used with the coolant density to express the weight of the coolant. By determining the thickness of the coolant layer for an optimum configuration the proper weight of the coolant can be found. The weight per square foot of insulation, structure, and coolant is

L 9 9 1

$$W = \rho_1 t_1 + \rho_2 t_2 + \rho_3 t_3 + w \tag{31}$$

where $w$ is the distributed weight per square foot of outside surface of the pump and piping equipment necessary to circulate the coolant. The pump and piping weight is assumed to vary linearly with the coolant weight; thus,

$$w = w_p + \epsilon \rho_3 t_3 \tag{32}$$

where $w_p$ is a fixed weight and $\epsilon$ is a constant of proportionality.

The substitution of equation (32) into equation (31) results in

$$W = \rho_1 t_1 + \rho_2 t_2 + \rho_3 t_3 (1 + \epsilon) + w_p \tag{33}$$

The coolant is assumed to protect the load-carrying structure by absorbing the heat transmitted through the structure by vaporization of the liquid. The result of this assumption is that the maximum structure temperature (which is taken as the design temperature) is the boiling temperature of the coolant. In this case, the problem is to minimize the weight for a fixed load, with respect to the variables, $t_1$, $t_2$, and $t_3$.

If the thermal capacity of the insulation and the structure is small compared to the latent heat of vaporization of the coolant, the heat-transfer problem is essentially one of quasi-steady state with all the heat passing through the insulation being absorbed by the coolant. When $T_{eq} < T_v$ no coolant is boiled. The conservative assumption is made that heat is absorbed only by the vaporization of the coolant. Thus, the heat-transfer equation may be written as

$$\frac{k_1}{t_1} \int_{\tau_j}^{\tau_k} \left(T_{eq} - T_v\right) d\tau = c_3 \rho_3 t_3 \tag{34}$$

where $\tau_j \leqq \tau \leqq \tau_k$ is the period during which $T_{eq} \geqq T_v$.

Equation (34) may be represented functionally as

$$q\left(t_1, t_3\right) = 0 = c_3 \rho_3 t_3 t_1 - k_1 \int_{\tau_j}^{\tau_k} \left(T_{eq} - T_v\right) d\tau \tag{35}$$

Equation (35) is the equation of constraint which is used in conjunction with equation (33) to minimize the weight.

When the Lagrange multiplier technique is applied to equations (33) and (35) there results

$$\frac{\partial W}{\partial t_1} + \psi \frac{\partial q}{\partial t_1} = 0$$

$$\frac{\partial W}{\partial t_2} + \psi \frac{\partial q}{\partial t_2} = 0$$

$$\frac{\partial W}{\partial t_3} + \psi \frac{\partial q}{\partial t_3} = 0$$

From equation (35) it is found that

$$\frac{\partial q}{\partial t_2} = 0$$

and therefore

$$\frac{\partial W}{\partial t_2} = 0$$

or that the extremum problem is independent of $t_2$. This result is to be expected because $t_2$ is in actuality dependent only upon the structure temperature, the imposed load, and the value of the temperature-dependent material property in the strength equation, all of which are prescribed and therefore invariant.

The remaining two equations yield

$$\frac{\partial W/\partial t_1}{\partial W/\partial t_3} = \frac{\partial q/\partial t_1}{\partial q/\partial t_3} \tag{36}$$

Taking the appropriate derivatives in equations (33) and (35) and substituting into equation (36) yields

$$\rho_1 t_1 = \rho_3 t_3(1 + \epsilon) \tag{37}$$

which is the optimizing relationship.

L
9
9
1

The weight per square foot of surface area becomes

$$W = 2\rho_1 t_1 + \rho_2 t_2 + w_p$$

$$= \rho_2 t_2 + 2\sqrt{\frac{k_1 \rho_1 (1 + \epsilon) G}{c_3}} + w_p \tag{38}$$

where $G = \int_{\tau_j}^{\tau_k} \left(T_{eq} - T_v\right) d\tau$ from equation (12a). Equation (38) represents the weight of an optimum structure subjected to any arbitrary variation in equilibrium temperature and is not restricted to the particular equilibrium-temperature variation used herein. Equation (38) also shows that the proportionality constant between the pump and piping-equipment weight and the coolant weight effectively reduces the thermal capacity of the coolant to a value which is

$$c_3' = \frac{c_3}{1 + \epsilon} \tag{39}$$

## Application of Mechanical Strength Criteria

Design on the basis of yield stress.- The optimum weight of a structure designed on the basis of either tensile or compressive yield will be determined. Equation (38) shows that the weight is linearly dependent upon the thickness of the primary structure. From equation (23)

$$t_2 = \frac{P}{b\sigma_y}$$

and therefore

$$W - w_p = \frac{\rho_2 P}{b\sigma_y} + 2\sqrt{\frac{k_1\rho_1(1 + \epsilon)G}{c_3}} \tag{40}$$

In equation (40) $\sigma_y$ is evaluated at the boiling point or evaporation temperature of the coolant. It should be noted that equation (40) is not restricted to a particular mode of time variation of $T_{eq}$ if $G$ is not so restricted.

L991

The weight of an insulated heat-sink structure may be compared to that of an insulated and cooled structure. Equation (40) can be expressed in terms of the load factor $\Omega_{y,e}$ as

$$\left(\frac{W - w_p}{b}\right)\left(\frac{Bb^2}{k_1\rho_1}\right)^{1/2} = \frac{\rho_2}{\sigma_y}\left\{1 + \frac{2\sigma_y}{\rho_2}\,\frac{\left[\frac{H(1 + \epsilon)}{c_3}\right]^{1/2}}{\Omega_{y,e}{}^{1/2}}\right\}\Omega_{y,e}{}^{1/2} \tag{41}$$

Since $H$ is a function only of $T_{eq,max}$ and $n$, the weight parameter can be plotted against $T_{eq,max}$ and $\Omega_{y,e}$ as in the case of the heat-sink configuration. Since the left-hand sides of equations (26) and (41) differ slightly because of $w_p$, minor arithmetical computation is necessary to compare weights directly unless $w_p = 0$. If $w_p = 0$, equations (26) and (41) can be compared directly.

<u>Design on the basis of plate buckling</u>.- The application of equation (27) (with $K = 4$) to the buckling criterion follows in a manner similar to that which was used for the heat-sink structure. From equation (27)

$$t_2 = \left[\left(\frac{P}{b^2}\right)\frac{3\left(1 - \mu^2\right)b^3}{\pi^2 E}\right]^{1/3} \tag{42}$$

Substitution of equation (42) into equation (38) yields

$$W - w_p = b\rho_2\left[\left(\frac{P}{b^2}\right)\frac{3\left(1 - \mu^2\right)}{\pi^2 E}\right]^{1/3} + 2\sqrt{\frac{k_1\rho_1(1 + \epsilon)G}{c_3}} \tag{43}$$

Equation (43) is manipulated to compare with the insulated heat-sink structure expressed by equation (30). Equation (43) then becomes

$$\left(\frac{W - w_p}{b}\right)\left(\frac{Bb^2}{k_1\rho_1}\right)^{1/2} = \frac{\rho_2}{\left[\frac{\pi^2E}{3(1 - \mu^2)}\right]^{1/3}}\left\{1 + 2\,\frac{\left[\frac{\pi^2E}{3(1 - \mu^2)}\right]^{1/3}\left[\frac{H(1 + \epsilon)}{c_3}\right]^{1/2}}{\rho_2\Omega_{b,e}^{1/2}}\right\}\Omega_{b,e}^{1/2} \tag{44}$$

and, as before, the weight per square foot of structure can be compared directly by comparing values of the weight parameters when $w_p = 0$.

L991

## COMPUTATIONS

Structural weight and loading parameters have been computed which permit a limited comparison to be made between an insulated heat-sink structure and an insulated and cooled structure. The computations were made to determine whether or not significant differences exist between the efficiencies of the two configurations.

In order to obtain comparative results it was necessary to assign certain characteristics and values to the parameters which determine the configuration weights. Equilibrium-temperature variations corresponding to boost-glide or reentry flight paths as given in equation (1) were used with $n = 2$. This temperature variation with time is shown in figure 5. Various values of the ratio $A/B^2$ were chosen so as to vary the maximum equilibrium temperature. (See eqs. (3).)

The parameters for the insulated and cooled structure were considered first. It was necessary to choose a cooling fluid. Several coolants were considered briefly, and water was chosen on the basis of availability, simplicity of handling equipment, and high latent heat of vaporization. The water was assumed to boil at atmospheric pressure to limit the design pressure of the coolant passage to one atmosphere. Thus, the coolant boiling temperature was fixed at 212° F. For this operating temperature 2024-T3 aluminum alloy was chosen as a familiar and practical structural material. Somewhat arbitrarily, the proportionality constant between the pump and piping weight and the coolant weight was taken as

$$\epsilon = 0.072$$

(which is the same as taking $c_3' = 900$ Btu/lb in eq. (39)).

With these chosen values equation (44) was used to compute the weight parameter for the insulated and cooled structure for the buckling criterion. The results are shown in figure 8 where the weight parameter is plotted as a function of $T_{eq,max}$ and $\Omega_{b,e}$.

L
9
9
1

The 2024-T3 aluminum alloy was also chosen for the structure material in the heat-sink case even though, if efficiency were the only consideration, the results of reference 2 indicate that HK31A magnesium alloy would be better. Aluminum was chosen to permit a more direct comparison between the heat-sink and cooled configurations (and because, in considering present fabrication practices, aluminum seems to be favored over magnesium for structural members near the external surface of a heated aircraft). Again equilibrium-temperature variations given by equation (1) with $n = 2$ were used. For the heat-sink structure successive values of the maximum equilibrium temperature (and hence $A/B^2$) were chosen and then, for each value, a range of $B/\alpha$ was assumed. The critical time and the maximum structure temperature were determined from equations (9) and (10). The load parameter was then computed from equation (29) and is plotted as a function of maximum equilibrium temperature and $B/\alpha$ in figure 9. This figure, together with figure 7 and equations (9) and (10) to determine the maximum structure temperature, was used with equation (30) to compute the weight of the insulated heat-sink configuration. This weight is plotted in figure 10 in the form of the weight parameter as a function of $T_{eq,max}$ and $\Omega_{b,e}$.

In addition to the previous computations the effect of varying the boiling point of the water was examined. The necessary increase in the weight of the coolant channel with an increase in pressure was neglected. In general, it is better to boil water at the highest possible temperature to obtain a slightly larger total heat absorption and to reduce the temperature gradient through the insulation. This gain was slight, however, and probably would not offset the increased weight of the coolant channels in an actual design. It should be mentioned that when other factors such as the internal environment of the space vehicle are considered the best overall efficiency may be obtained by vaporizing the water at room temperature ($75^\circ$ F). Although this may not be optimum from the strict structural-efficiency viewpoint, operation at this low temperature eliminates the need for internal cooling equipment to provide a habitable temperature.

## RESULTS AND DISCUSSION

### General

The relative weights of the insulated heat sink and the insulated and cooled configurations with aluminum used as the structural material

may be observed by comparing figure 10 with figure 8. This comparison is limited to cooled structures where the weight of the pumps and piping may be considered proportional to the coolant weight, with a factor of proportionality of 0.072. For these calculations there was very little difference between the weights of the two configurations; in fact, for the range of $\Omega_{b,e}$ considered for the heat-sink structure, figure 10 may be superposed upon figure 8. As $\Omega_{b,e}$ decreases the insulated and cooled configuration becomes more efficient.

It is worth noting that for the insulated heat-sink configuration the weight of the primary structure is always greater than the weight of the insulation in an optimum design. This fact is shown by the partial differential equation (eq. 22) which defines the necessary relationships for an optimum structure. The condition which determines the optimum for the insulated and cooled configuration is that the weight of the insulation is equal to the weight of the coolant when the coolant weight is calculated by using the reduced thermal capacity. (See eqs. (32), (37), and (39).)

L
9
9
1

There is a significant difference between the relationships that exist for the optimum heat-sink configuration and those for the optimum internally cooled configuration. The equations which determine the optimum insulated heat-sink structure (eqs. (25) and (29)) contain the applied load P, whereas the equation for the internally cooled structure (eq. (37)) is independent of the load. In the former case the amount of heat sink depends upon the plate thickness, and, because the heavier loads require thicker plates, an increased amount of heat sink is more readily available. For the internally cooled configuration the structure temperature was prescribed so that the amount of absorbable heat became independent of the thickness of the primary structure. Thus, the optimization equation (eq. (37)) relates only the insulation weight to the coolant weight and is independent of the applied load. The simplification introduced by prescribing the structure temperature is impossible for configurations such as the heat-sink configuration if a true optimum is sought.

A similar but more subtle point arises for internally cooled configurations when several different coolant vaporization temperatures are under consideration. In general, the coolants will have different, but prescribable, vaporization temperatures. Because the structure thickness $t_2$ depends upon the temperature-dependent material properties, $t_2$ will vary with the coolant vaporization temperature chosen. The problem of finding the coolant which provides the least total weight of structure now depends upon $t_2$. However, if each coolant and its vaporization temperature is considered independently the equations developed herein are applicable, and the optimum weights for each coolant can

be found. A comparison of these optimum weights (by directly comparing the values obtained for the weight parameters) will reveal the most efficient coolant.

As was mentioned in the section entitled Computations, the slight increase in thermodynamic efficiency realized by boiling the coolant (water) at a pressure higher than atmospheric pressure would probably be offset by the increase in the weight of the coolant passage required to contain the higher pressures, and, if overall vehicle efficiency be considered, the total vehicle weight might be minimized by vaporizing
L the water at $75^{\circ}$ F to eliminate the need for heavy internal climate-
9 conditioning equipment.
9
1

Use of the Computed Curves

All of the computed curves contained herein are based upon the thermal environment described by equation (1), with $n = 2$. As was mentioned previously, curves for other heating conditions may be computed by the procedure outlined at the end of the section entitled Computations.

Once a set of curves has been obtained for the desired value of $n$, the optimum protection system may be found for any combination of $A$ and $B$. The first design step is to determine the type of insulation to be used. The optimum insulating material will be that which, first of all, will withstand the peak value of the equilibrium temperature $T_{eq,max}$, and second, will have the minimum value of $k_1\rho_1$. It is obvious that this insulation will be an optimum for both the heat sink and the internally cooled configurations, whichever may later turn out to be the better method of designing the structure. The values of $B$, $b$, $P$, and $k_1\rho_1$ are used to compute the loading parameter $\left(\Omega_{y,e}\right.$ or $\Omega_{b,e}$, whichever may apply$\left.\right)$. The structural configuration of least weight (heat sink or internally cooled) may be obtained from a comparison of the weights determined from charts such as figures 8 and 10.

Under the assumption that the weight comparison, together with other considerations (such as simplicity of design) indicates a choice of the heat-sink configuration, it is then necessary to find the values of $t_1$ and $t_2$ for a minimum-weight structure. The value of $B/\alpha$ for optimum conditions can be found from a figure similar to figure 9. Since $B$ is known, $\alpha$ is determined; $B\tau_{cr}$ may be found from equation (10); and the maximum temperature of the primary structure $T_{2,max}$ can be determined from equation (9). Once $T_{2,max}$ is known, either $\sigma_y$ or $E$ (whichever is applicable) is known; and, since the design load is prescribed,

the thickness of the primary structure $t_2$ is determined from either equation (23) or equation (27). Because $\alpha = \frac{k_1}{c_2\rho_2t_2t_1}$, $t_1$ can easily be computed.

On the other hand, if an internally cooled structure is used, either $\sigma_y$ or $E$ is immediately fixed by the coolant vaporization point, and $t_2$ may be readily computed. The times at which the vaporization of the coolant commences and ends are found from equation (11). By combining equation (34) with equation (37),

L991

$$\frac{c_3(1 + \epsilon)}{k_1\rho_1}(\rho_3t_3)^2 = \int_{\tau_j}^{\tau_k} (T_{eq} - T_v)d\tau$$

from which $t_3$ may be computed. This value is used to find $t_1$ through equation (37).

## CONCLUDING REMARKS

The Lagrange multiplier technique has been used to determine the design conditions for optimum insulated heat-sink structures and for optimum insulated and cooled structures. This method is advantageous over the direct method used in the references when the equilibrium temperature at the outside surface of the insulation is other than a very simple function of time.

Computations were made to illustrate the application of the analysis to a thermodynamic flight path similar to that of atmospheric reentry. The computations were restricted to aluminum load-carrying structures. Except under conditions favorable to the insulated heat-sink design (high structure loads and short heating periods) where the weights of the two configurations are equal, water-cooled structures are more efficient than heat-sink structures.

The results of the calculations are presented in graphic form by means of load parameters which account for the loading condition, heating condition, and insulation properties. The load parameters are similar to those derived in previous investigations for the simpler case of constant temperature at the outside surface of the insulation.

The differential equations show the general result that, for optimum heat-sink structures, the weight of the primary structure always exceeds

the insulation weight. For the internally cooled structure an optimum configuration exists when the insulation weight equals the combined weight of the coolant, pump, and piping if the pump and piping weight is considered to vary in linear proportion with the amount of coolant required; the weight of the primary structure does not influence this optimizing relationship.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Air Force Base, Va., December 13, 1961.

L
9
9
1

## REFERENCES


1. Davidson, John R.: Optimum Design of Insulated Tension Members Subjected to Aerodynamic Heating. NASA TN D-117, 1959.

2. Davidson, John R., and Dalby, James F.: Optimum Design of Insulated Compression Plates Subjected to Aerodynamic Heating. NASA TN D-520, 1961.

3. Harris, Robert S., Jr., and Davidson, John R.: An Analysis of Exact and Approximate Equations for the Temperature Distribution in an Insulated Thick Skin Subjected to Aerodynamic Heating. NASA TN D-519, 1961.

4. Timoshenko, S.: Strength of Materials. Part II - Advanced Theory and Problems. Third ed., D. Van Nostrand Co., Inc., c.1956.

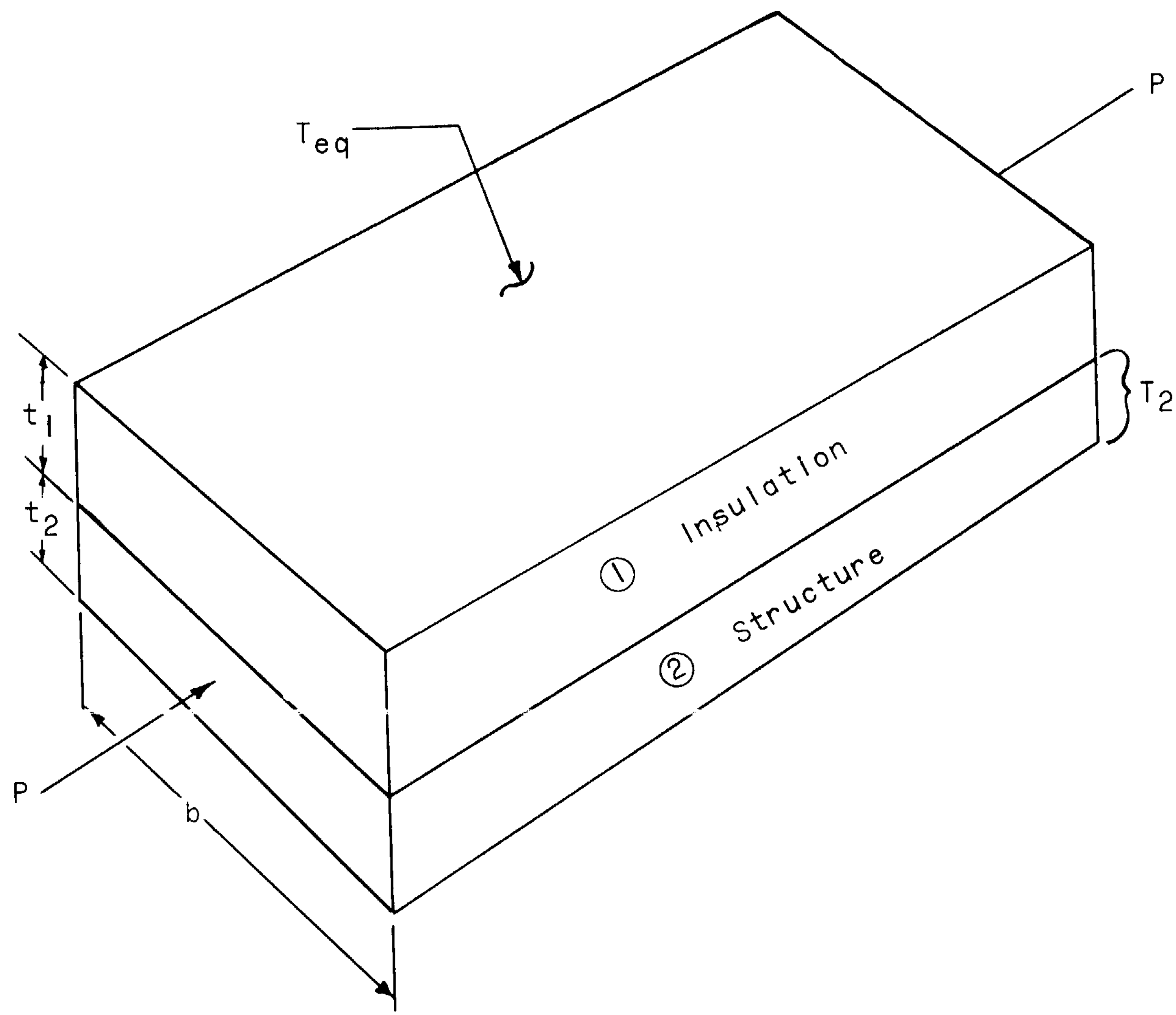


Figure 1.- Sketch of insulated heat-sink structure loaded in compression.

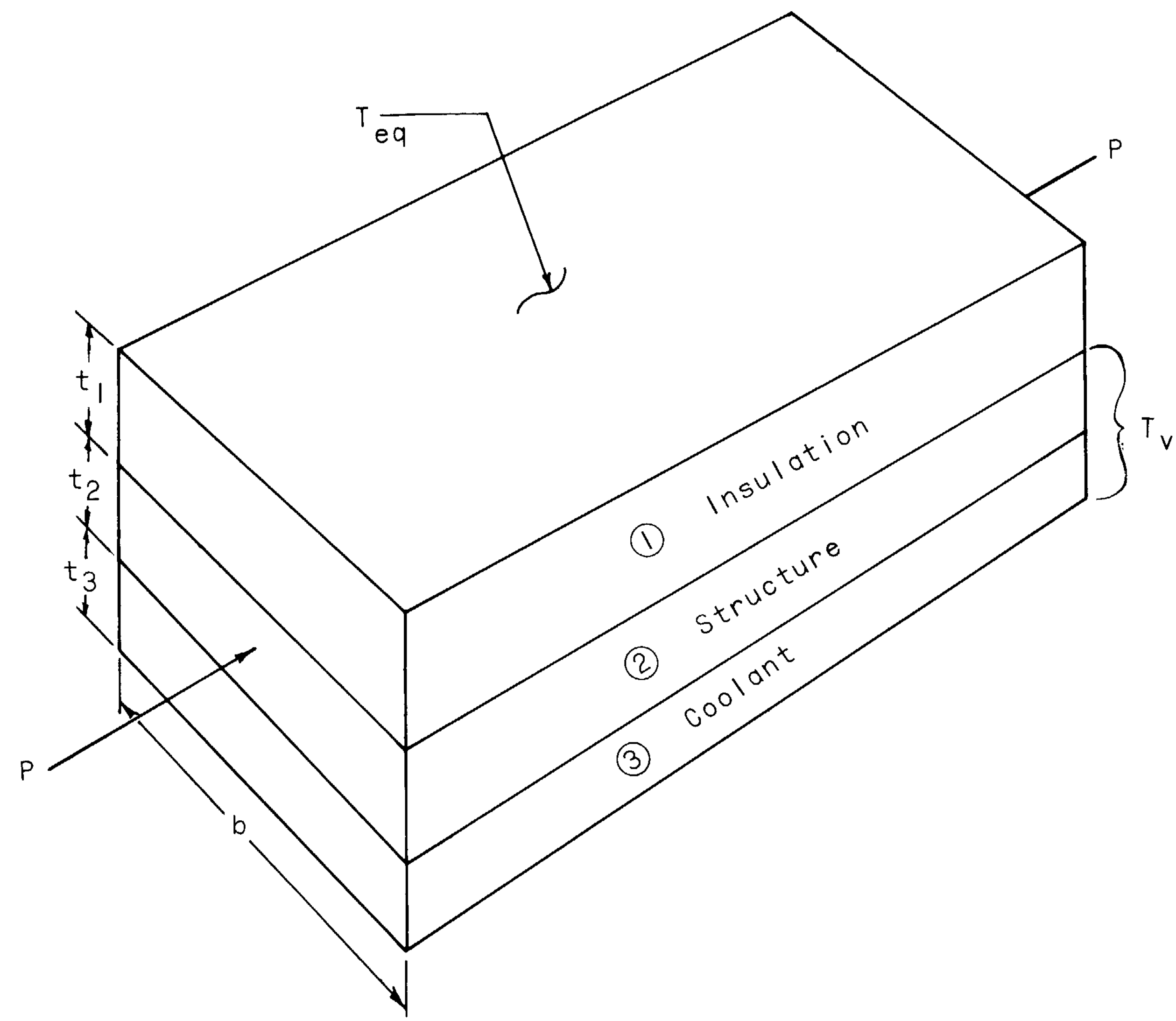


Figure 2.- Sketch of insulated and cooled structure.

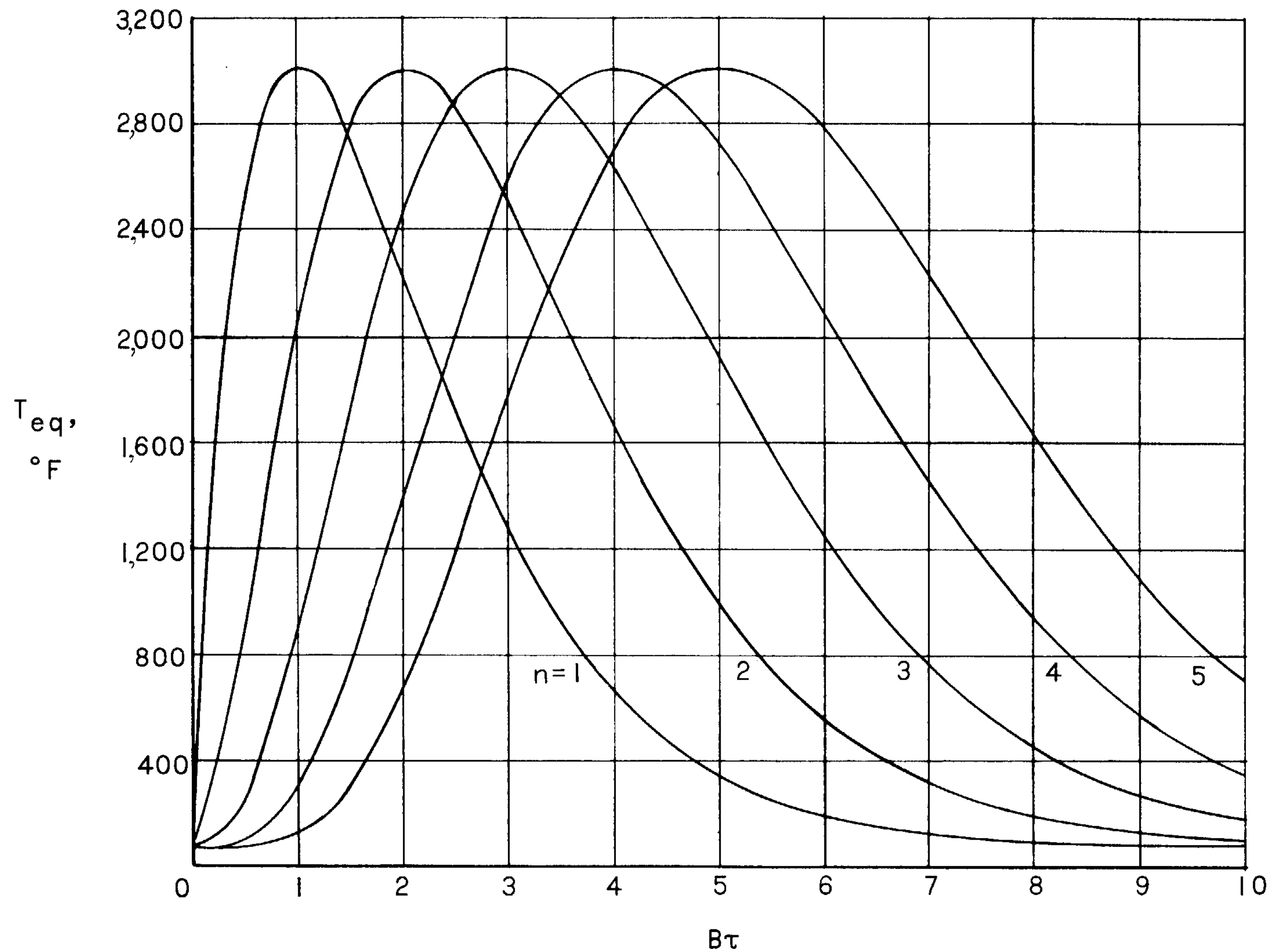


Figure 3.- Equilibrium-temperature variation with time computed from $T_{eq} - T_o = A\tau^n e^{-B\tau}$ for various values of n and for $T_{eq,max} = 3{,}000^\circ$ F.

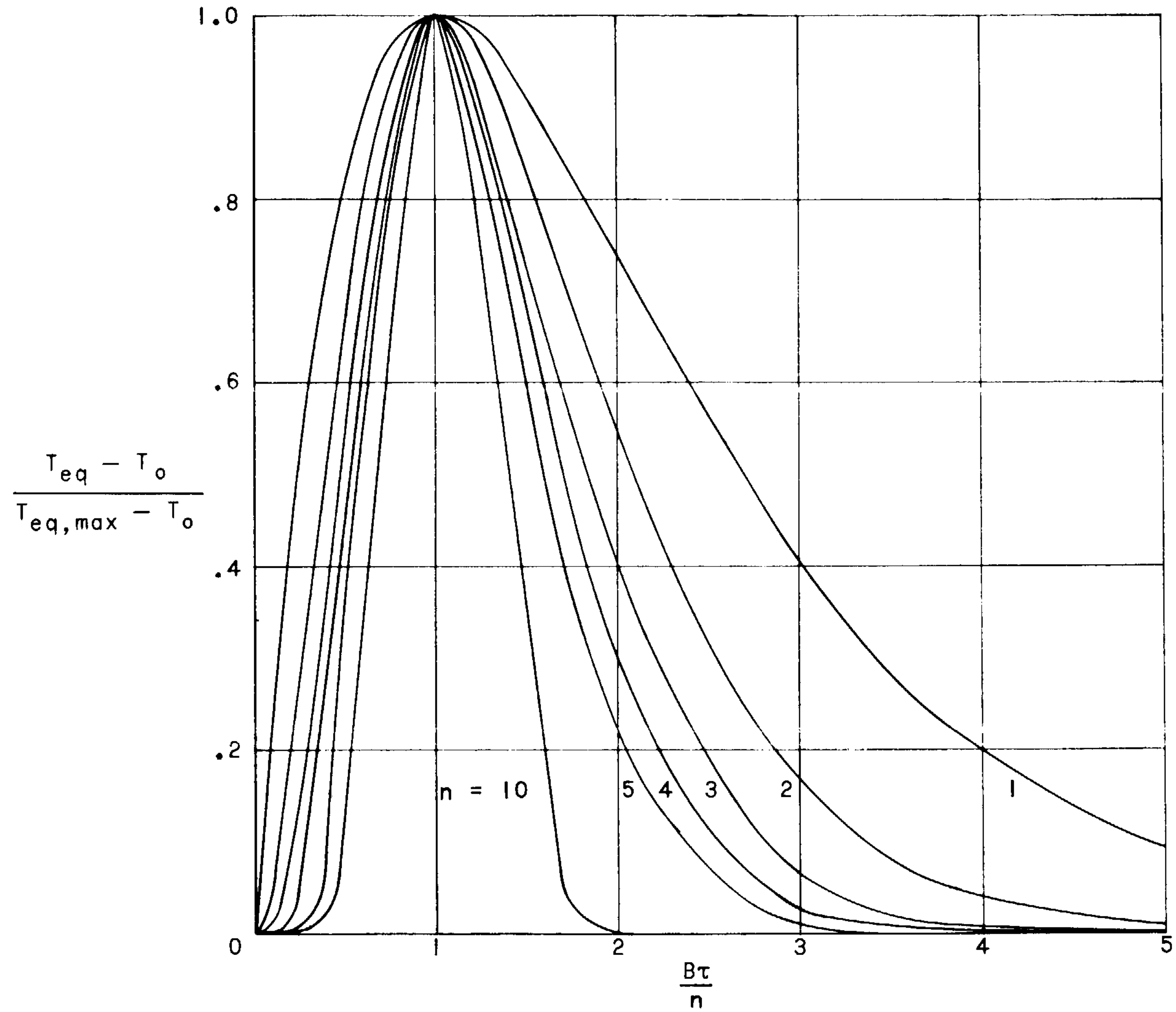


Figure 4.- Equilibrium-temperature variation with time plotted in dimensionless form for various values of $n$.

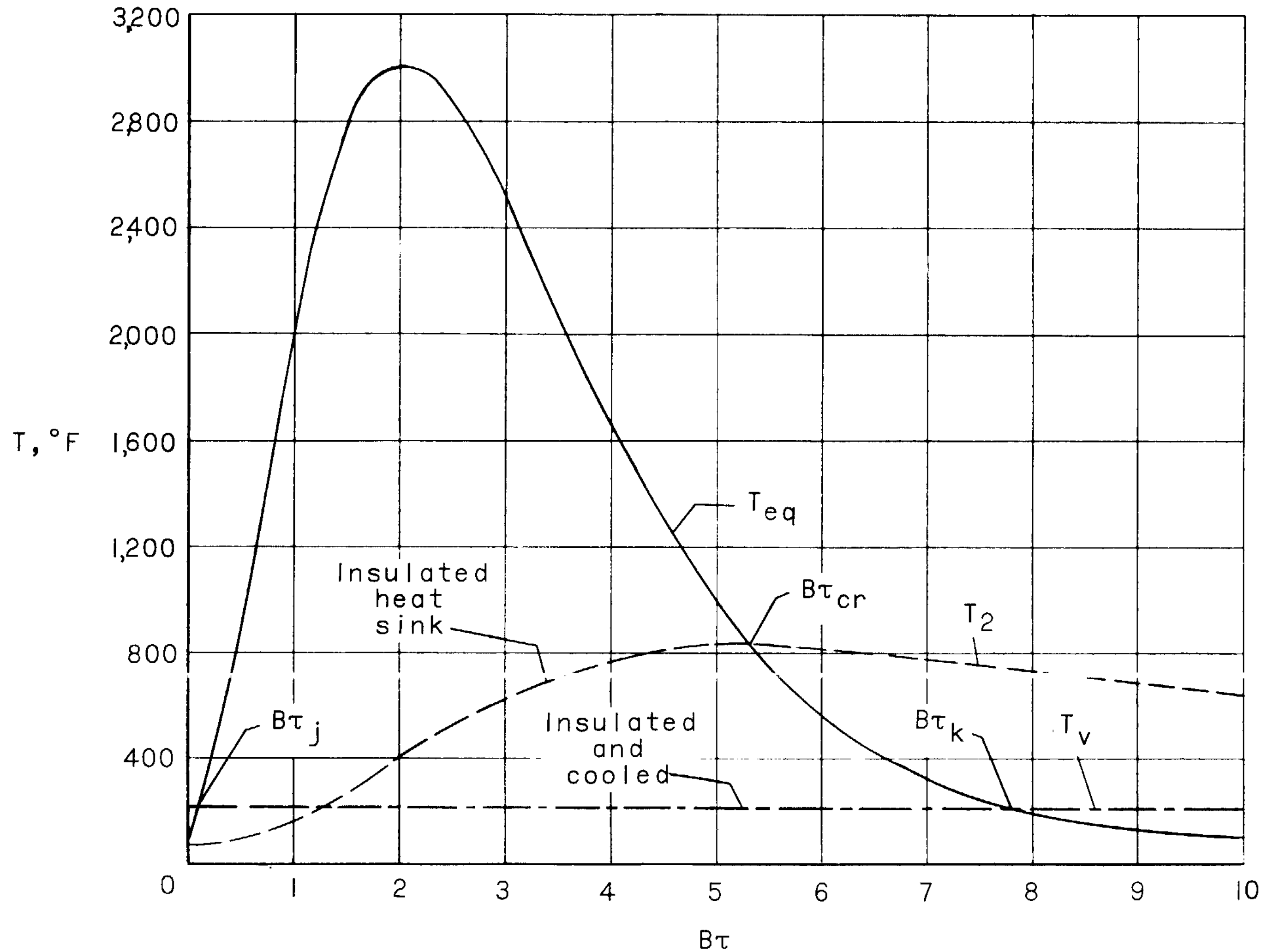


Figure 5.- Plot of equilibrium temperature, primary-structure temperature, and coolant boiling temperature for $n = 2$ and $T_{eq,max} = 3,000^{\circ}$ F.

L-991

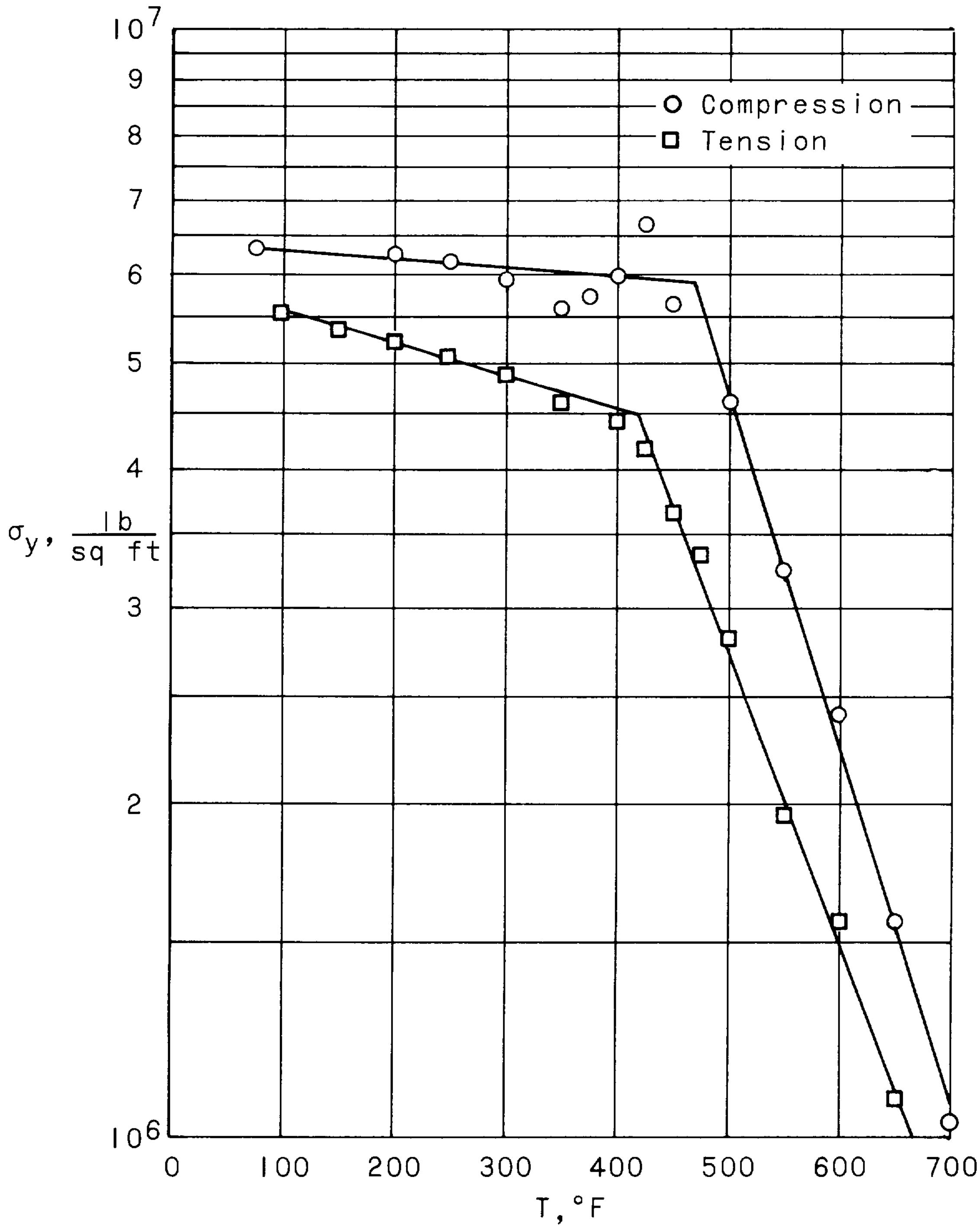


Figure 6.- Yield stress as a function of temperature for 2024-T3 aluminum. Solid lines express the analytic approximation to experimental data.

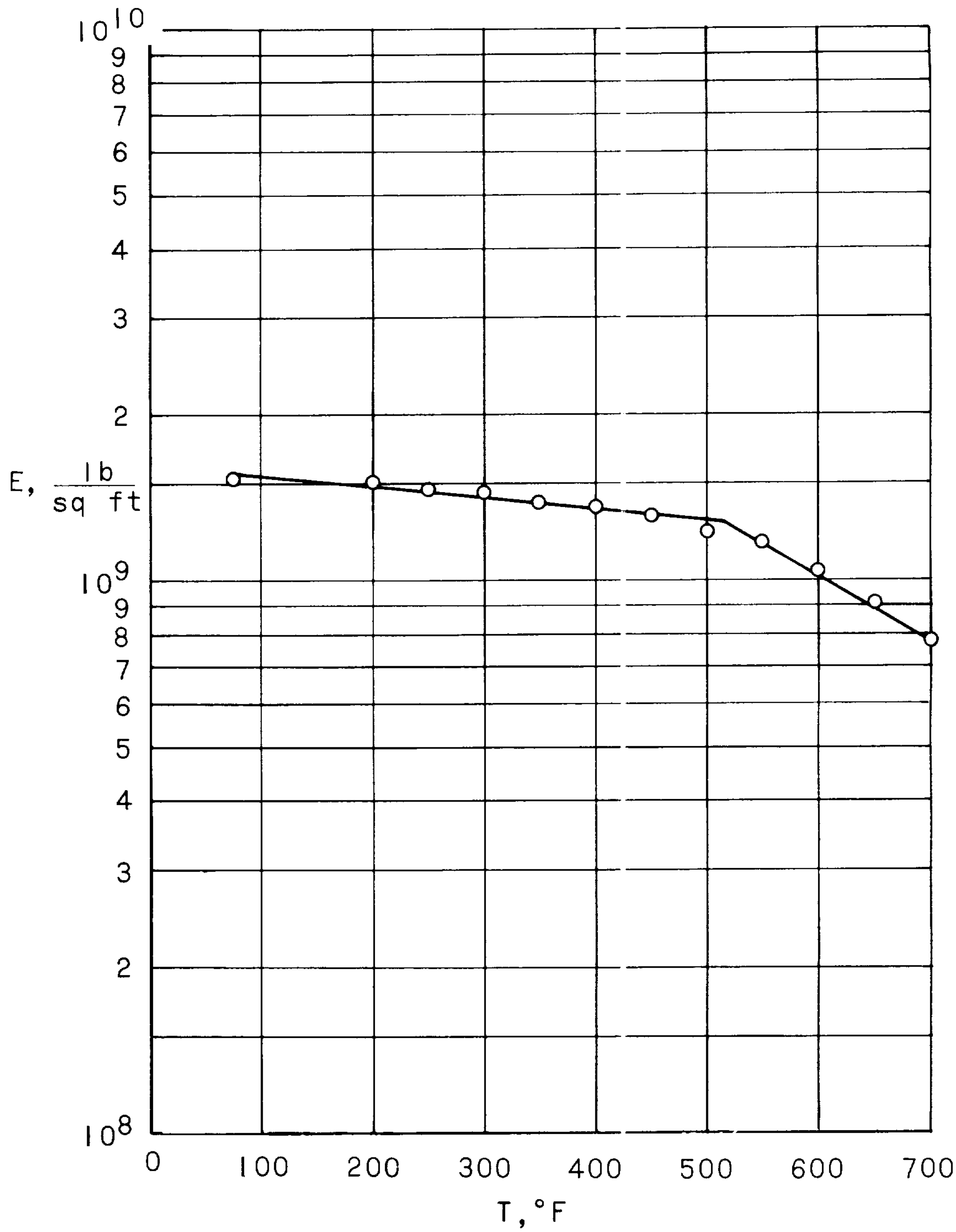


Figure 7.- Modulus of elasticity as a function of temperature for 2024-T3 aluminum. Solid line expresses the analytic approximation to experimental data.

L-991

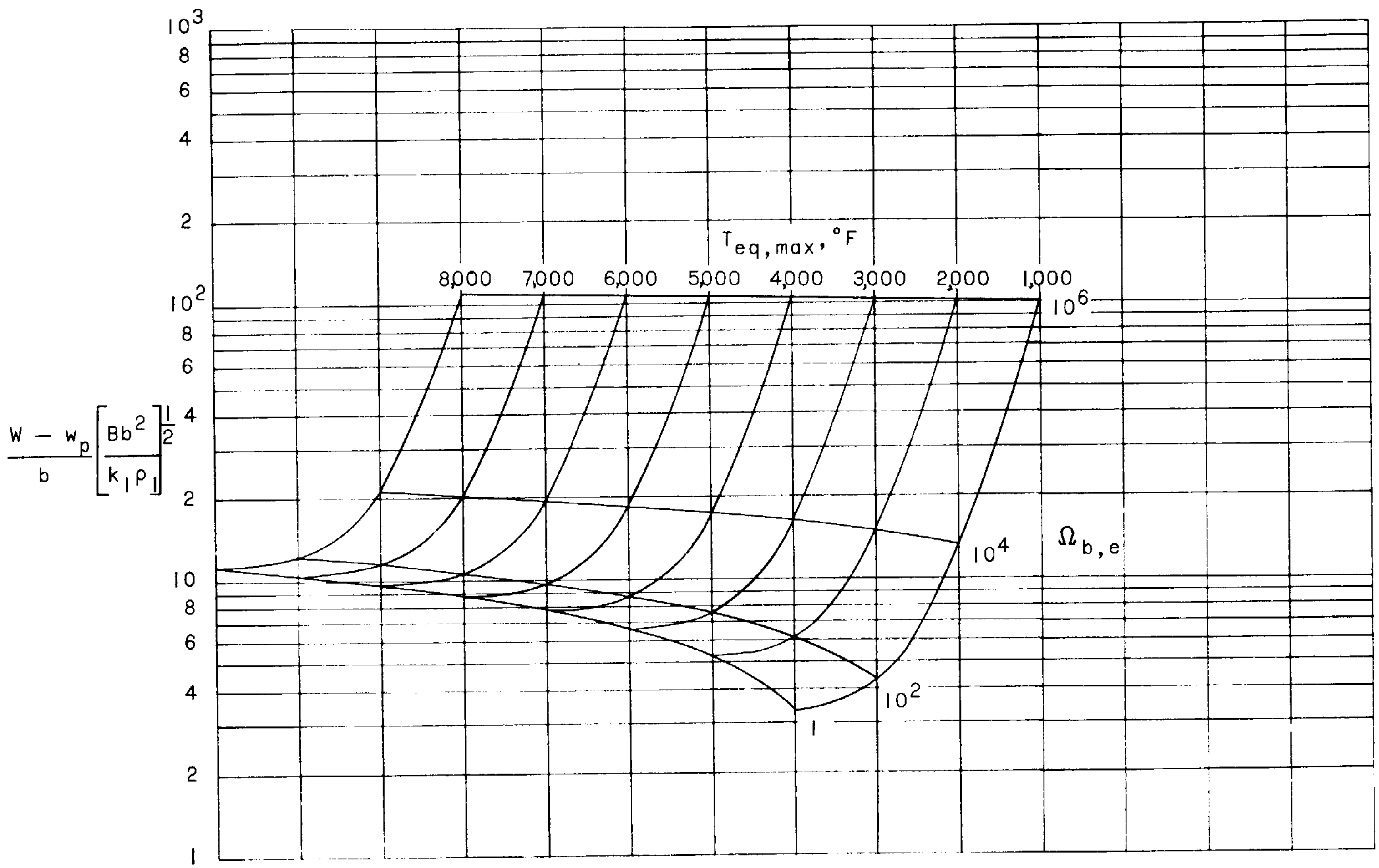


Figure 8.- Weight parameter for optimum insulated and water-cooled aluminum structures with $\epsilon = 0.072$ and a vaporization temperature of 212° F.

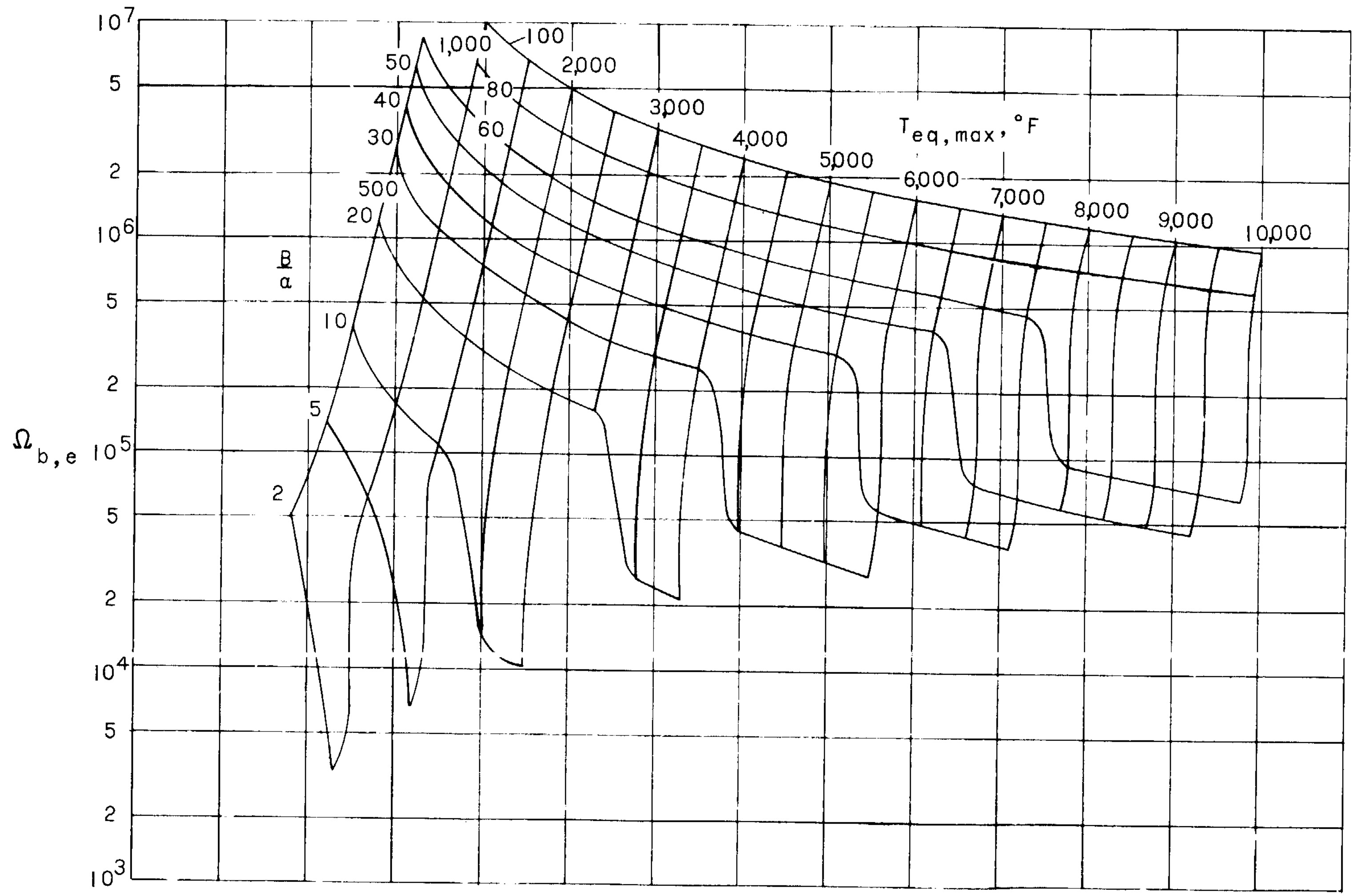


Figure 9.- Load parameter $\Omega_{b,e}$ for insulated heat-sink aluminum structures plotted against $T_{eq,max}$ and $B/\alpha$ for an initial temperature of 75° F.

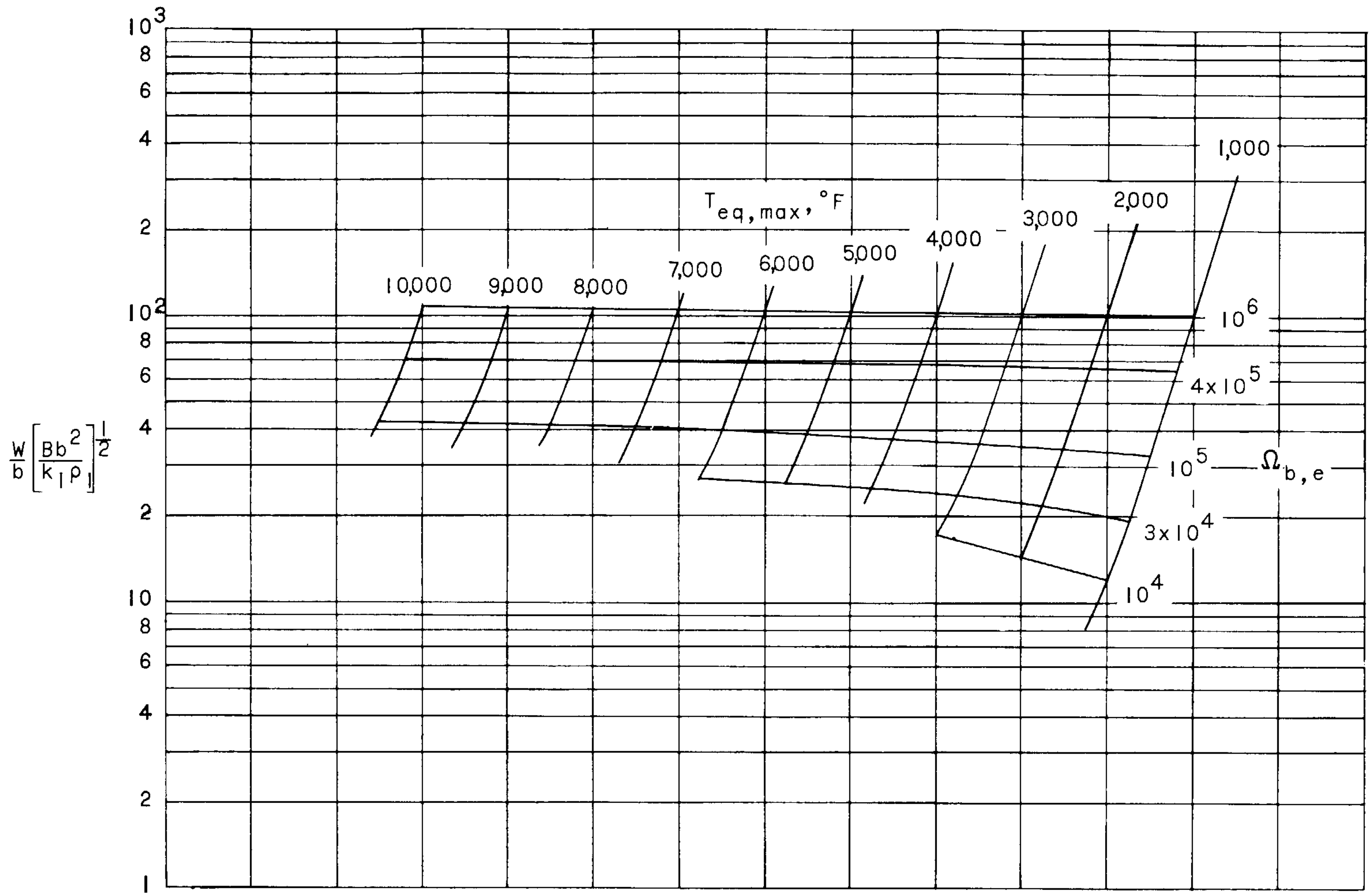


Figure 10.- Weight parameter for optimum insulated aluminum plates designed on the basis of buckling plotted against $T_{eq,max}$ and $\Omega_{b,e}$ for an initial temperature of 75° F.



NASA-Langley, 1962 L-991